

Growth of a Polynomial not Vanishing in a Disk

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Abstract— A well known result due to Ankeny and Rivlin states that if $P(z) = \sum_{j=0}^n a_j z^j$ is a polynomial of degree n with no zeros in $|z| < 1$, then for $R \geq 1$,

$$\max_{|z|=R} |P(z)| \leq \frac{R^n + 1}{2} \max_{|z|=1} |P(z)|.$$

Recently, Jain obtained a generalization of the above inequality by considering polynomials with no zeros in $|z| < k$, $k \geq 1$ and has simultaneously taken into consideration the s th derivative ($0 \leq s < n$) of the polynomial, instead of the polynomial itself and proved

$$\max_{|z|=R} |P^s(z)| \leq \frac{1}{2} \left\{ \frac{d^s}{dR^s} (R^n + k^n) \right\} \left(\frac{2}{1+k} \right)^n \max_{|z|=1} |P(z)|, \quad \text{for } R \geq k.$$

In this paper, we generalize as well as improve upon the above inequalities and other related results.

Keywords: Polynomial, Maximum Modulus Principle, Zeros.

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Reference

- [1] . C. Ankeny and T. J. Rivlin, *On a theorem of S. Bernstein*, Pacific J. Math., **5** (1955), 849 – 852.
- [2] A. Aziz and Q. Aliya, *Growth of polynomials not vanishing in a disk of prescribed radius* Int. J. Pure Appl. Math., (**41**)(2007), 713 – 734.
- [3] N. K. Govil, M. A. Qazi and Q. I. Rahman, *Inequalities describing the growth of polynomials not vanishing in a disk of prescribed radius*, Math. Ineq. Appl., **6** (2003), 453 – 467.
- [4] V. K. Jain, *A generalization of Ankeny and Rivlin's result on the maximum modulus of polynomials not vanishing in the interior of the unit circle*, Turk. J. Math., **931**(2007), 89 – 94.
- [5] G. V. Milovanovic, D. S. Mitrinovic and Th. M. Rassias, *Topics in polynomials, Extremal problems, Inequalities, Zeros*, World scientific, Singapore, 1944.